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STUDY ON APPLIED TECHNOLOGIES TO PROPOSE SOLUTIONS FOR ENHANCING HYDRAULIC FRACTURING EFFICIENCY IN TIGHT SANDSTONE RESERVOIRS OF THE CUU LONG BASIN

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Summary

The main objective of this study is to evaluate hydraulic fracturing technologies applied in the oil fields of the Cuu Long basin to obtain lessons learned and propose solutions to improve the efficiency of hydraulic fracturing methods for the tight clastic formations therein. For that purpose, an assessment of representative hydraulic fracturing cases in oil production wells is conducted, focusing on well candidate selection, lithological composition, and reservoir rock properties for calculating geomechanical parameters and stress fields. Geomechanical models that simulate fracture development are compared to actual applications, namely in terms of effective treatment radius, required fracturing injection pressure, proppant size, injection pressure control processes, as well as flow return and well monitoring methods. The study also reviews modern and effective hydraulic fracturing technologies currently applied worldwide, particularly advanced geomechanical simulations, to accurately predict fracture networks, permeability, conductivity, the length, width, and height of fractures within productive fracturing zones. Based on the research outcome, solutions are proposed to improve the efficiency of hydraulic fracturing for the Miocene and Oligocene formations in the Cuu Long basin.

Key words: Hydraulic fracturing, fracking, geomechanical model, fracture prediction model, proppant, fracturing fluid, oil incremental, tight Oligocene formation, tight sandstone reservoir.

1. Introduction

Currently, oil and gas industry remains an irreplaceable and essential resource for global economic development, including in Vietnam. In recent years, oil and gas production in Vietnam has shown a strong declining trend as major oil fields in the Cuu Long basin - such as Bach Ho, Rong, Su Tu Den, Te Giac Trang, Rang Dong, and Hai Su Den oil field have entered the late stages of their production lifecycle. Newly discovered fields are mostly small reserves or located at edges of structures, with limited in-place reserves, or they belong to tight Oligocene formations with challenging production conditions, making development difficult and insufficient to offset declining production. Detailed production forecasts for Vietnam's oil fields through 2035 predict a rapid decline, with output expected to fall to only 10 million tons per year after 2024 and below 2 million tons per year by 2035 [1].

To counter this decline in production, oil field operators have implemented various well interventions to enhance oil recovery of the producing wells. Such interventions are especially crucial for maintaining and increasing production to meet annual targets, given the limited number of new wells being drilled, declining field output, and the need for economically viable technical solutions amidst low oil and gas prices [2]. As a result, well intervention techniques like add-perforation, acidizing, water/gas shut-off, hydraulic hracturing, and electric submersible pump installations have been applied in Cuu Long sandstone basin, particularly in tight Oligocene formations. These measures have significantly contributed to increasing production and recovery in these fields [3, 4].

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In Vietnam, hydraulic fracturing applications in the Cuu Long basin have primarily focused on the Miocene and Oligocene formations. The basement formation has been applied to a limited extent due to its natural fractures, which cause severe fluid loss during fracturing and prevent achieving the necessary pressure differential for effective fracturing [2]. Therefore, implementing production enhancement methods in existing wells or newly production wells in tight sedimentary formations, such as the Oligocene, is now a pressing issue.

The initial part of this study focuses on analyzing and assessing the current methods used for hydraulic fracturing, identifying technical uncertainties in well selection and design. Due to limited assessments of the geomechanical properties in the selected well area and various risks in evaluation, the well selection process is analyzed based on geological characteristics, reservoir permeability, appropriate permeability ranges, lithology, geomechanical parameters, stress fields, and geomechanical modeling for hydraulic fracturing to predict fracture aperture, fracture volume, fracturing fluid, proppant diameter, proppant volume, fracturing pressure, and closure pressure. Technical controls are applied during pre-fracturing testing, fluid injection, deep placement of proppants within fractures, and closure pressure control to stabilize proppants in fractures, creating and maintaining new fracture channels that connect the well with the reservoir in targeted zones.

The study assesses post-fracturing effectiveness by monitoring fluid flow rates, production dynamics, and the process of returning the hydraulically fractured well to production. It analyzes typical wells for specific geological formations and technological stages, focusing on simulating fracture network patterns and hydraulic fracturing parameters for wells that have achieved significant oil production increases in the Cuu Long basin. Additionally, the study evaluates advanced and effective fracturing technologies used globally, particularly advanced geomechanical simulations for hydraulic fracturing, to accurately predict fracture networks, fracture permeability and conductivity, as well as fracture length, width, and height in targeted fracturing intervals. Based on these research findings, solutions are proposed to modernize and improve hydraulic fracturing efficiency for the Miocene and Oligocene formations in the Cuu Long basin.

2. Overview of hydraulic fracturing in the Cuu Long's sandstone

Curently, there are 26 oil fields in the entire Cuu Long basin, of which 20 fields are in production and the remaining 6 planned for development in the near future. Production is mainly concentrated in the Cuu Long basin from several large oil fields, such as Bach Ho, Rong, Rang Dong, Su Tu Den, which have been producing for around 20 years. The remaining producing fields are smaller in reserves. Hydraulic fracturing has been experimentally applied to various geological formations, including the basement, Lower Oligocene, Upper Oligocene, and Lower Miocene - particularly in areas with very poor reservoir permeability, wells with high contamination levels, and wells where production cannot be improved by conventional production methods or acid treatments. Hydraulic fracturing for sedimentary reservoirs in the Cuu Long basin has been extensively implemented in fields managed by VSP, such as Bach Ho, Rong, Ca Tam, and Kinh Ngu Trang [5], with the majority of projects concentrated in the first two listed (Table 1).

Table 1. *Hydraulic fracturing projects applied in Cuu Long sandstone reservoirs*

No.	Oil fields	Project number	Reservoir
1	Bach Ho	118 projects	Miocene, Upper Oligocene, Lower Oligocene
2	Ca Tam	6 projects (102, 104, 2X, 108)	Upper Oligocene, Lower Oligocene
3	Gau Trang	GTC1-1P	Miocene
4	Rong	21 projects (RP2, RP3, RC7, RC6, RC5, RC9)	Miocene, Upper Oligocene, Lower Oligocene
5	Tho Trang	ThTC1-2X	Miocene
6	Hai Su Den	HSD-4X	Oligocene
7	Te Giac Trang	TGT-15X	Oligocene
8	Lac Da Vang	LDV-2X, LDV-4X	Oligocene
9	Su Tu Nau	SNS-1P	Oligocene
10	Kinh Ngu Trang	KNT-1X	Oligocene
11	Block 01 & 02	RBA 14XP, Jade 4X, Emerald 2X	Miocene, Oligocene

Hydraulic fracturing in the Upper and Lower Oligocene accounts for nearly 77% of treatments, whereas in the Miocene, it comprises only about 21%. Fracturing is rarely performed in the basement formation, only 2%, due to geological and production characteristics that make this method unsuitable. The basement formation is rarely targeted because of its low effectiveness, mainly due to large natural fractures that lead to severe fluid loss during fracturing and an inability to create sufficient pressure differentials for effective fracturing.

The success of these methods depends on the technology used, implementation approach, reservoir properties, well characteristics, geological targets, and the specific characteristics of each oil and gas field. Hydraulic fracturing for oil fields of Cuu Long basin faces numerous challenges due to offshore production conditions and extreme operating environments. These include the significant depths of tight Oligocene formations (3,000 - 4,300 m), high temperatures (120 - 150°C), high pressures (250 - 400 bar), high clay content in lithology, poor permeability and connectivity, and high heterogeneity. For tight Oligocene targets, characteristics such as high abnormal reservoir pressure and low permeability (1 - 10 mD) are typical. Criteria for assessing hydraulic fracturing applications for this target are presented in Table 2.

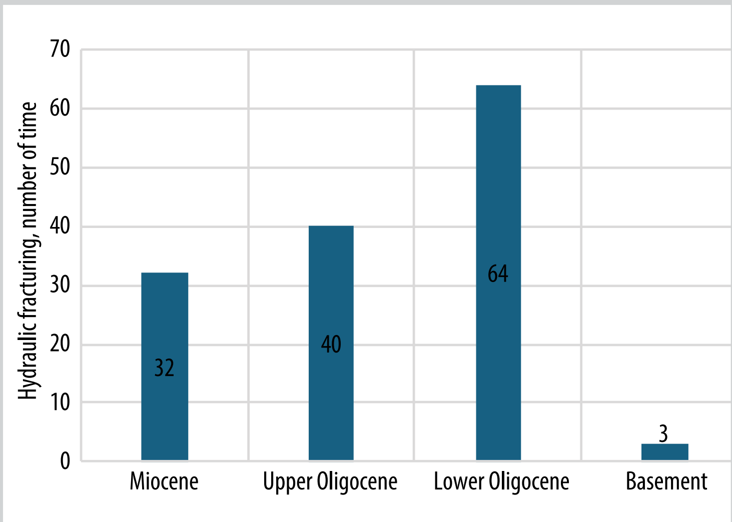

***Figure 1.** Statistics of hydraulic fracturing projects for reservoirs of Cuu Long basin.*

According to statistics on hydraulic fracturing projects in Vietnam, the effectiveness of hydraulic fracturing projects evaluated for each target shows that the Miocene formation has a success rate of approximately 69%, the remaining 31% being less effective. For the Oligocene formation, the application effectiveness is better, with a success rate of around 76%.

3. Simulation studies for hydraulic fracturing forecasting and application effectiveness in the Cuu Long basin

The selection and application of hydraulic fracturing methods on production wells targeting tight Oligocene formations in the Cuu Long basin are significantly affected by substantial technical and economic risks due to various factors. To analyze the reasons behind the successes and failures of hydraulic fracturing projects, it is essential to fully evaluate the three main stages of the fracturing process under the production conditions of wells in the Cuu Long basin, as follows:

- Well candidate selection: This involves analyzing geological characteristics, reservoir engineering parameters, production rates, near-wellbore permeability, geomechanical stress fields, and historical production data. Additionally, well structures and technical issues, such as casing, cement quality, and downhole equipment, are considered to assess wells for treatment.

- Design of the hydraulic fracturing process and well preparation: This stage includes assessing the geomechanical model, reservoir and rock properties to conduct in-depth

***Table 2.** Criteria for Oligocene reservoirs*

No.	Criterion	Commentary
1	Geographical location	Regional proximity to offshore Vietnam
2	Formation pressure, anomaly coefficient	Over 300 atm, anomaly coefficient 1.1 - 1.7
3	Reservoir	Oligocene, Miocene (second priority)
4	Composition of rocks	Sandstones, clays, siltstones
5	Depth of reservoirs	Over 3,000 m
6	Permeability	0.5 - 10 mD
7	Formation thickness	From 50 m
8	Porosity	Up to 20%

analyses for selecting proppant volume and size, additives, and pressure parameters based on stress fields. It also involves designing fracturing options/technologies, pump rates, pressure control, and injection fluid volume, as well as preparing wellbore equipment for hydraulic fracturing operations.

- Implementation of hydraulic fracturing in stages: This phase includes performing a minifrac test, injecting fracturing fluid, and pumping proppant to stabilize fractures and create new channels that connect the well with the reservoir. After fracturing, well effectiveness is assessed by an evaluation program that monitors fluid flow rates, production dynamic parameters, and observes the process of putting the fractured well back into production.

Several wells have been successfully hydraulically fractured thanks to the correct selection of wells and the accurate assessment of geomechanical parameters, lithology, reservoir permeability, and well productivity. This process enabled the construction of a fracture simulation model to determine the fracture width, volumetric distribution in various directions within the reservoir, permeability, and fracture conductivity. As a result, the correct proppant size and amount were selected, along with the appropriate volume and properties of the fracturing fluid. The pumping pressure and closure pressure were accurately calculated to optimize fracture creation and maintain long-term stability of the proppants as well as the conductivity of the oil channel from the reservoir to the well. The paper presents two typical hydraulic fracturing projects successfully applied to the tight Oligocene formation of the Cuu Long basin.

3.1. Case study 1: Well BO-01

The BO-01 well, located in the Oligocene formation, has a production interval from 3,790 - 3,940 m, with an effective reservoir thickness of approximately 36 m. The average porosity is around 14%, and the average permeability is approximately 1.3 mD. The lithology composition, represented by a QFR diagram (quartz-feldspar-rock fragment), shows that quartz comprises about 55 - 65%, typically the dominant component that helps achieve a low Young's modulus while maintaining high material strength. Feldspar makes up around 25 - 30%, contributing to the overall hardness but keeping the Young's modulus at a low to medium level. Rock fragments account for approximately 10 - 15%, contributing minimally to strength but helping maintain a low Poisson's ratio, which aids in controlling deformation and elasticity.

The well has been in operation since 2014 with an initial production of around 40 sm^3/day, but it quickly dropped to 10 sm^3/day within about 3 years of operation. The well was assessed to have potential for increased production due to the significant remaining oil reserves in the area and the very low water cut in the well or the surrounding area.

Based on the criteria for selecting wells for hydraulic fracturing, well BO-01 was chosen for in-depth studies on geomechanics, lithology, and the development of a hydraulic fracturing simulation model to predict fracture aperture, proppant size, proppant volume, as well as fracturing pressure and closure pressure. The design included pre-fracturing test, pumping process for hydraulic fracturing, injection of proppants deep into the fractures, and controlling the closure pressure to stabilize the proppants in the fractures, creating and maintaining new fracture networks connecting the well to the reservoir and the affected area of the well. An evaluation program was developed to assess the effectiveness of the fracturing treatment, monitor fluid production rates, production dynamics, and oversee the process of bringing the hydraulically fractured well back into production.

Geomechanical parameters such as stress field, Young’s modulus, Poisson’s ratio, and borehole logs were incorporated into the hydraulic fracturing simulation model for analysis. The results of the comparison between the simulated and

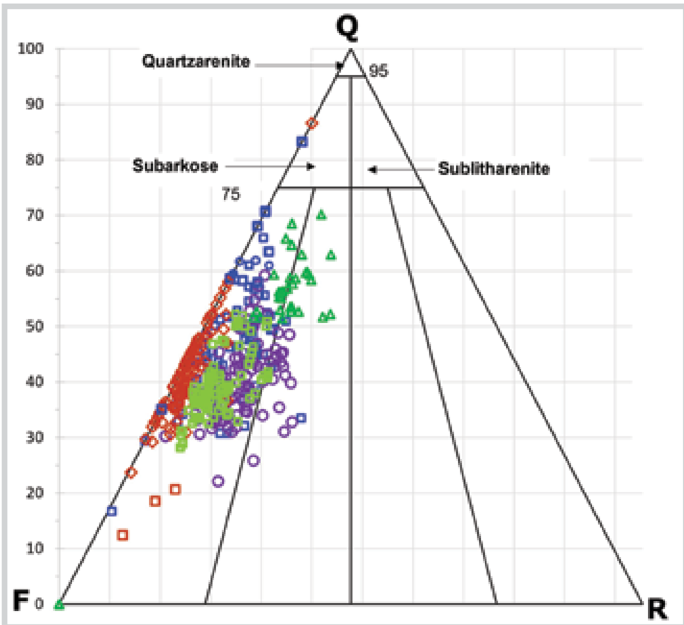

Figure 2. *Lithological composition characteristics of the Oligocene formation.*

actual hydraulic fracturing outcomes are presented in the figure below. The results show that the differences between the design and actual results are not significant, particularly with respect to fracture volume, where the actual fracture length, height, and width are all larger than those in the initial simulated design (Table 3).

The results show that the effectiveness of the pumping process is better than expected. 131 tons of proppant were pumped into the created fractures, the pump pressure was raised to 540 atm, and the pumping time lasted for 3 hours.

The production after hydraulic fracturing in well BO-01 shows that the it increased from 10 sm^3/day to over 80 sm^3/day, then decreased and stabilized at a production rate above 20 sm^3/day for the next 2 years of operation. The cumulative oil production, as shown in Figure 5, demonstrates the effectiveness of the hydraulic fracturing method for well BO-01.

3.2. Case study 2: Well BO-02

Hydraulic fracturing was performed in the Oligocene layer at a depth range of 3,880 - 4,040 m, with an effective reservoir thickness of about 32 m and permeability ranging from 0.5 to 1.2 mD. The production behavior of surrounding wells in the area showed high production rates initially (>100 sm^3/day), but it declined rapidly after about 1 - 2 years of operation. During the later stages of decline, the wells sustained low but stable production rates of around 10 - 15 sm^3/day over 5 - 7 years. Well BO-02 was also designed and subjected to pumping according to the prescribed design procedure. The fractures were evaluated based on actual data and re-simulated, as shown in Figure 6.

***Table 3.** Comparison of the design and actual results of hydraulic fracturing application*

Parameters	Simulation design	Actual implementation
Propped fracture length (m)	56.8	62.8
Propped fracture height (m)	88.5	110.7
Average propped fracture width (cm)	0.55	0.86
Fracture conductivity (mD.m)	525.9	454.4
Fcd	3.1	9
Proppant volume (tons)	136.5	131.1

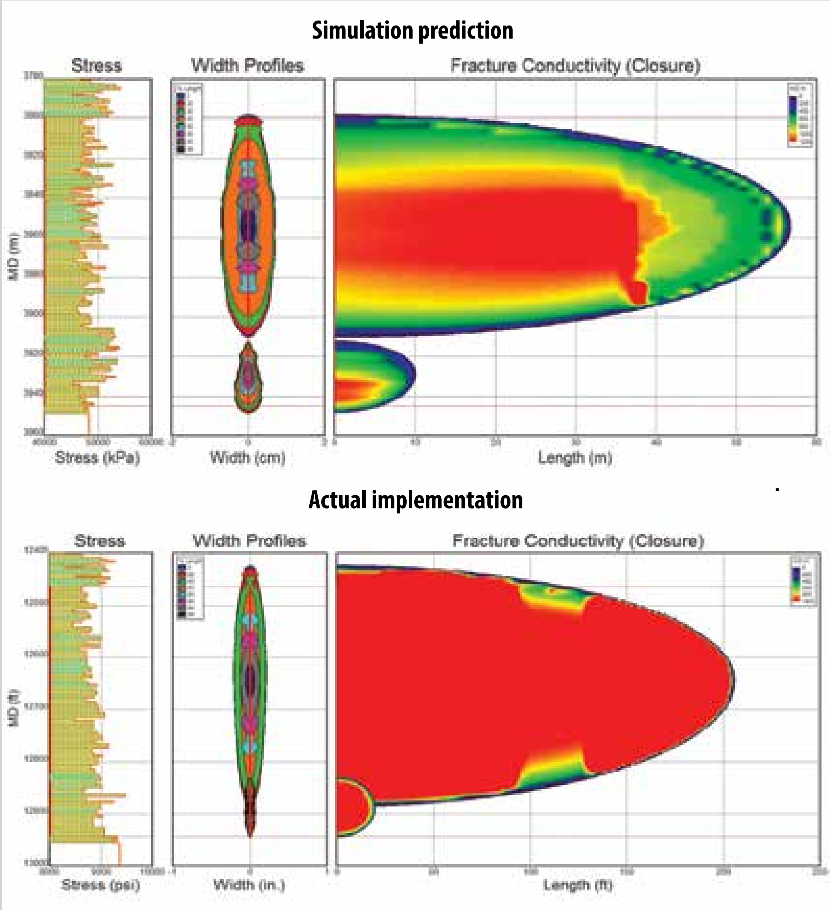

***Figure 3.** Evaluation of the accuracy of the hydraulic fracturing model and practical application.*

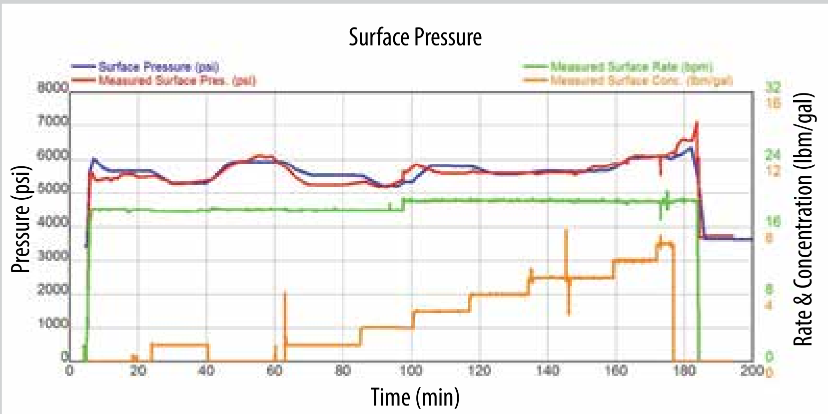

***Figure 4.** Graph of pressure monitoring during the hydraulic fracturing pumping process.*

Figure 5. *Production of well BO-01 before and after hydraulic fracturing.*

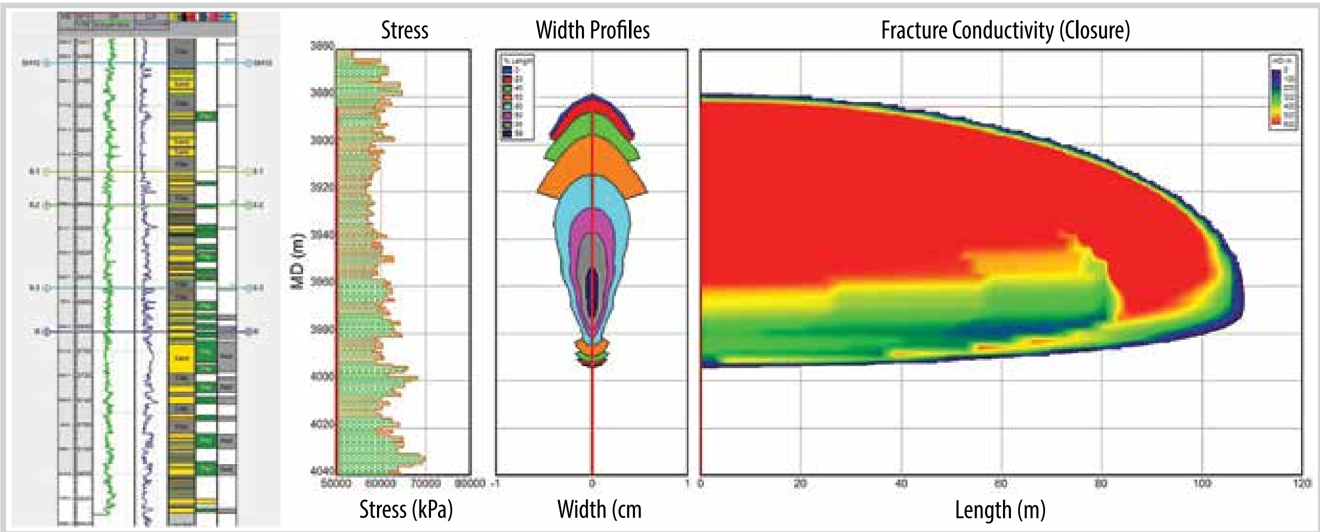

Figure 6. *Simulation of the actual fractures during the hydraulic fracturing implementation.*

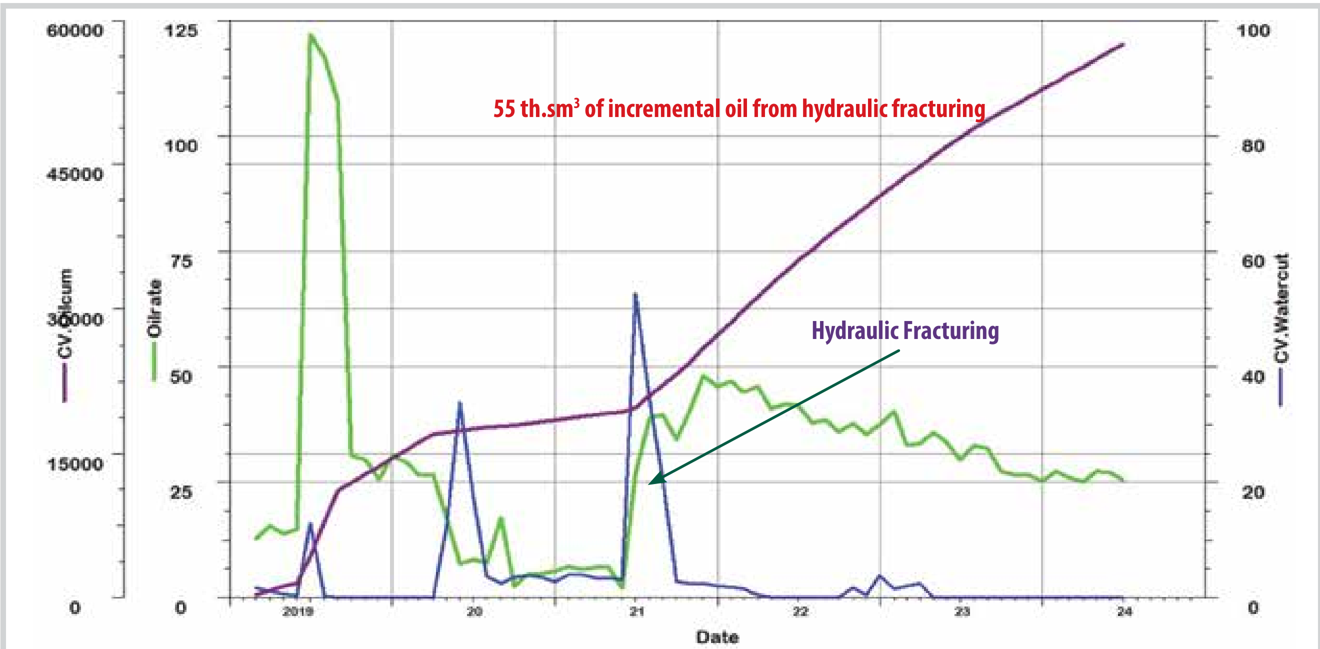

Figure 7. *Flow rate and increased oil production after hydraulic fracturing in the tight Oligocene formation.*

The production rate increased from 5 sm^3/day to over 45 sm^3/day, then gradually declined and maintained an average of over 25 sm^3/day for nearly 3 years of operation. The total oil increase from hydraulic fracturing to the present time has reached 55 thousand sm^3. The hydraulic fracturing project has been highly successful both technically and economically, contributing to increased production across the entire field.

4. Evaluation of technologies and geomechanical models that can be appropriately applied to the Cuu Long basin

The typical hydraulic fracturing cases above have demonstrated the effectiveness of applying this process to oil wells in the clastic reservoirs of the Cuu Long basin. Application procedures must be followed in selecting wells with suitable lithology for fracturing, ensuring that the remaining oil volume in the reservoir is sufficient to sustain production through the created fractures. The injection process must control fracture pressure according to design and maintain the appropriate injection rate. The proppant concentration must be injected with precise volume and timing to ensure fracture stability as closure pressure is reduced, as shown in cases BO-01 and BO-02. Based on the applied research steps and hydraulic fracturing technology in Vietnam, as well as the evaluation results from over 150 hydraulic fracturing projects implemented for the Miocene and Oligocene targets in the Cuu Long basin, several causes for success or failure can be summarized as follows:

- Failure causes: One possible cause of failure could be the inaccurate geological assessments, particularly the geomechanical evaluations of the selected wells. The reservoir structure, stress field of the formation according to the well depth, may not have been studied in detail or may not match the actual conditions of the selected well, leading to incorrect calculations of the required pumping pressure and the fracturing fluid systems used to create effective fractures. Therefore, the geomechanical model plays a crucial role in providing parameters to predict the length, width, height, and permeability of the created fractures. This allows for detailed calculations on the effective area of hydraulic fracturing, proppant size, fracture fluid volume, and proppant-carrying fluid volume to be pumped, as well as the decision on the appropriate hydraulic fracturing technology for the well [9].

- Another cause of failure may arise from non-optimal evaluations and application procedures, leading to several issues affecting the well being treated, such as the inability to bring the parker to the surface due to deformation of the production tubing caused by improper fracturing pressure and the influence of the stress field in the formation.

- The effectiveness of hydraulic fracturing technology could also be influenced by situations where treatments were applied in wells with multiple product layers but without adequate evaluations of the well structure, equipment, or appropriate technology. In such cases, when fracturing was performed in the lower perforated interval, communication with the upper perforated interval occurred, causing fluids to migrate upward through the casing outside the production tubing due to poor quality cementing, leading to treatment failure.

- One of the causes of failure in well treatment is the selection of unsuitable proppant fluids or the non-optimized fracturing pumping process. The pumping process did not manage to inject the full calculated volume of proppant-carrying fluid into the fractures, resulting in a large volume of proppant remaining in the production tubing. This required implementing multiple solutions after the treatment to release the parker in the well, such as using coiled tubing, pulling the tubing out of place, or perforating the production tubing to flush it, leading to a failed treatment. Additionally, in some wells, the treatment was carried out without pumping fracturing fluid to create fractures, and instead, proppant-carrying fluid was directly pumped into the well. In this fracturing method, the proppant-carrying fluid serves both to open and develop the fractures and to stabilize them, preventing the fractures from closing. However, this method carries significant risks, such as proppant getting stuck in the pumping column due to failure to pump the full amount of proppant-carrying fluid as designed, forcing the pump to stop when the pressure suddenly rises above the limit (screen out), especially in wells with a high loss coefficient, and it may not be effective. Therefore, nowadays, most hydraulic fracturing operations are carried out in a fully sequential manner: first, high-pressure fracturing fluid is pumped to create fractures, then a continued injection of fluid containing proppant and proppant-carrying fluid follows, followed by the injection of chemicals and additives to push the proppant deep into the formation through the fractures and maintain fracture stability after the treatment.

Based on the research output and the adoption of advanced technologies worldwide, and by comparing with the current hydraulic fracturing technology applied in Vietnam, the following technologies and geomechanical modeling approaches can be appropriate to the tight sandstone formations of the Cuu Long basin:

- Hydraulic fracturing models: With the reservoir conditions of high pressure and temperature, low permeability (0.01 - 100 mD), and multiple sandstone layers in the reservoir, geomechanical models combined with fluid flow simulations are suitable.

+ P3D model (Pseudo-3D) [6]: It is suitable for simulating the shape and size of fractures in reservoirs with low permeability. The model accounts for factors that limit the vertical and horizontal extent of fractures, helping predict the fracture propagation in formations with permeability ranging from low to medium.

+ Planar 3D model: It can simulate the development of fractures in a 3D space, both horizontally and vertically. This model is useful when dealing with reservoirs with multiple sandstone layers, where fracture propagation needs to be precisely controlled to ensure efficient hydrocarbon extraction from different layers.

+ Coupled geomechanical-fluid flow models with DFN (discrete fracture network) simulation [7]: This model helps calculate the interaction between rock and fracturing fluids, which is crucial for dealing with reservoirs that have high temperature and low permeability. It allows the optimization of fracture propagation while maintaining pressure stability and fluid flow in the reservoir. The DFN model simulates systems of both natural and induced fractures, aiding in the optimization of oil and gas flow through fracture networks in multi-layered reservoirs.

- Fracturing fluids and proppants suitable for high temperatures in the Cuu Long basin:

+ High-temperature stable fracturing fluids [8]: Fracturing fluids such as polymer-based gel systems or cross-linked gel can be used. Cross-linked gel helps stabilize the structure of the fluid and maintain the necessary viscosity at high temperatures, thereby keeping the proppant in the fractures for longer periods.

+ High-temperature, high-pressure proppants: In high-temperature environments like the Cuu Long basin, it is necessary to use specialized proppants with high thermal resistance, such as ceramic proppants or bauxite proppants with advanced plastic coating technology. These proppants offer excellent strength and thermal resistance, ensuring they do not break under high-temperature and high-pressure conditions.

- Effective multi-stage fracturing technology for multi-layered sandstone reservoirs: The method divides the well into multiple segments and performs hydraulic fracturing in each segment, aiming to maximize the contact area with the reservoir and extract oil and gas from different layers.

- Technology for high flow rate and high-pressure control: Given the high-temperature and high-pressure conditions, a high-flow high-pressure control system is necessary, such as sliding sleeve or plug-and-perf technologies. These systems help regulate and optimize the hydraulic fracturing process for each segment of the well while maintaining the required flow rate and pressure for each wellbore section.

5. Conclusions

The hydraulic fracturing process in sandstone oil fields requires strict adherence to standards, from well selection and design to construction and long-term maintenance. Lessons learned from real-world failures show that each stage can significantly impact the overall effectiveness of a hydraulic fracturing project. By applying the solutions derived from these experiences, it is possible to optimize the fracturing process and increase extraction efficiency.

A thorough assessment of geological characteristics, lithological composition, geomechanical parameters, rock stress fields, reservoir permeability, and remaining in-place oil reserves within the well's influence radius is crucial to select the appropriate well, build an accurate fracture simulation model, and implement the fracturing design using the optimal pumping process.

For geological targets with poor and tight properties currently being exploited in the Cuu Long basin, such as the Oligocene formations of the Bach Ho, Rong, and Ca Tam fields, or potential targets like Te Giac Trang, Kinh Ngu Trang, and Lac Da Vang, it is essential to consider the application of new technologies. These include geomechanical modeling combined with flow simulation for fracture modeling, using fracturing fluids and proppants with optimal size and properties suited for the high temperatures of the Cuu Long basin. Effective technologies such as multi-stage fracturing for multi-layered sandstone reservoirs should be applied to achieve significant production increases without technical risks during the fracturing process.

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